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Reliability of Circular (*n, f, k*) System with Weighted Components

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Abstract— *This paper deals with the reliability of circular weighted-(n, f, k): G and circular weighted-(n, f, k): F systems with the application of Universal Generating Function. The (n, f, k): F (G) system consists of n components ordered in a line or a cycle, and it fails (works) if and only if, there are at least f failed (working) components or at least k consecutive failed (working) components. In this study we impose some conditions on weight and compare the reliability of these models. This study is also extended to evaluate the sensitivity of the considered systems. Some numerical examples are presented to illustrate the proposed method.*

Keywords*: Consecutive weighted-k-out-of-n systems, weighted-(n, f, k) system, system reliability, Universal Generating Function (UGF), sensitivity.*

1. INTRODUCTION

In various reliability problems the contribution made by the components are considered to be identical. But in many real life situations contribution of system's components are different to the system and the dependency of system's failure is not only on the failed components but also on their individual contribution. In such a case, the reliability of the system cannot be defined only in terms of its structure but also in terms of weight attached with the components in the structure. This situation is closely related to the system with weighted components.

A *k*-out-of-*n* system model with unequal weight for the components is introduced by Wu and Chen [11]. This system is named as weighted–*k*-out-of-*n*: *F* (*G*) system. The weighted *k*-out-of-*n*: *F* (*G*) system consists of *n* components each having its own positive integer weight, fails (works) iff the total weight of failed (working) components is at least *k*. Clearly these systems are equivalent to the *k*-out-of-*n* system if each component has weight 1. Later, Negi and Singh [8] evaluated the reliability of the non-repairable complex system by using weighted *k*-out-of-*n* system as subsystems. Wu and Chen [12] evaluated the reliability of consecutive-weighted-*k*-out-of-*n*: *F* system. These systems fail iff the total weight of the failed consecutive components is at least *k*. Eryilmaz and Tutuncu [4] studied the reliability of linear consecutive-weighted-*k*-out-of-*n* independent and nonidentical system having in homogeneous markov dependent components. Lambris and Papastaviridis [6] provided an exact formula to evaluate the reliability of linear & circular consecutive *k*-out-of-*n*: *F* system. Li and Zuo [7] also done remarkable work in the field of weighted-*k*-out-of-*n* systems.

The (*n, f, k*): *F* (*G*) system consists of *n* components and the system fails (works) iff there are at least *f* failed (working) components or at least *k* consecutive failed (working) components (Chang et al. [1]; Sun and Liao [9]; Tung [10]). These systems are useful in automatic payment system in bank (Sun and Liao [9]) and in the evaluation of furnace system (Zuo and Wu [13]). Cui et al. [3] studied (*n, f, k*) systems. In 2005, they [2] discussed a general system consisting of *N* modules with the i^{th} module composed of n_i components in parallel in which the system fails iff there exists at least *f* failed components or at least *k* consecutive failed modules. They obtained the reliability for both linear and circular cases. Also Eryilmaz and Aksoy [5] introduced and studied the reliability of linear (*n, f, k*): *F* and (*n, f, k*): *G* systems with weighted components which are represented as linear (*n, f, k,* ***w***): *F* and (*n, f, k,* ***w***): *G*. They proposed recursive formulas for computing the reliability of these systems. But there are some useful systems like circular (*n, f, k*): *F* (*G*) systems with weighted components whose reliability is never obtained in the past with the application of Universal Generating Function (UGF). So the main focus of this paper is to study these systems and obtained the reliability of the same with the application of UGF.

Keeping the above discussed matter in view, we focus our present study on reliability estimation of circular (*n, f, k*): *F* and (*n, f, k*): *G* systems with weighted components. These systems are represented as circular (*n, f, k,* ***w***): *F* and (*n, f, k,* ***w***): *G*. UGF approach is presented for the reliability (unreliability) and sensitivity evaluation of these systems. An example is taken to illustrate the proposed method. We impose some conditions on the weights and compare reliability of these systems in different conditions.

2. NOTATIONS AND DEFINITIONS:

2.1. Notations: The following notations have been used in this paper.

Table 1: Notations

n	the number of components in the system
gi	the state of the component *i* (i.e. *gi* = 1(0) iff component *i* is working (failed))
Pr	Probability
pi	probability related to the working state of the component *i*
wi	the weight of the component *i*
w	(*w*1, *w*2,.....,*wn*), represents the weight vector
CRw (*n*, *f*, *k*: *F*)	the reliability of circular weighted-(*n*, *f*, *k*, ***w***): *F* system
CFw (*n*, *f*, *k*: *G*)	the unreliability of circular weighted-(*n*, *f*, *k*, ***w***): *G* system

2.2. Definitions:

2.2.1. Circular weighted-(*n*, *f*, *k*, *w*): F system: A circular weighted-(*n*, *f*, *k*, ***w***): F system consists of *n* components ordered in a cycle and the system fails iff the total weight of failed components is at least *f* or the weight of the consecutively failed components is at least *k*.

2.2.2. Circular weighted-(*n*, *f*, *k*, *w*): G system: A circular weighted-(*n*, *f*, *k*, ***w***): G system consists of *n* components ordered in a cycle and the system works iff the total weight of the working components is at least *f* or the total weight of the consecutively working components is at least *k*.

Clearly these models can be reduced to the usual (*n*, *f*, *k*) models if ***w*** = (1,1,......,1). As an example consider a lighting system consists of *n* circularly arranged light bulb having different wattage, gives different importance (weight) to the bulbs within the system. A certain level of illumination must be achieved for the lightning of the system and this is not only depending upon the total number of failed (working) components but also on the consecutively failed (working) components.

3. RELIABILITY EVALUATION BY USING UNIVERSAL GENERATING FUNCTION (UGF)

UGF is introduced by I. Ushakov in 1986 and extended by A. Lisniansky and G. Levitin. It is an efficient technique to evaluate reliability of different systems which extends the concept of moment generating function. It reduces the computational complexity of Multi- state system (MSS) reliability assessment.

The u-function (UGF) of an independent discrete random variable *X* is defined as a polynomial

$$U_X(z) = \sum_{k=1}^{K} q_k z^{x_k}$$

where the variable *X* has *K* possible values $x_1, x_2, ..., x_k$ and q_k is the probability that *X* is equal to x_k and z is any variable. This means that a one-to-one correspondence exists between the probability and UGF of a discrete random variable (r.v.).

Consider *n* independent discrete random variables $X_1, X_2, ..., X_n$. Let the UGF of r.v. $X_1, X_2, ..., X_n$ be $U_1(z), U_2(z), ..., U_n(z)$ respectively and *f* ($X_1, X_2, ..., X_n$) an arbitrary function. Then by employing composition operator $\otimes$ the UGF, $U_f(z)$ of function *f* ($X_1, X_2, ..., X_n$), can be obtained as:

$$U_f(z) = \otimes(U_1(z), U_2(z), ..., U_n(z)) .$$

Now let there are *n* independent components. g_i denotes the states of the i^{th} component and $g_i = 1$ implies that component *i* is in working state while $g_i = 0$ implies that component *i* is in failed state, where $i = 1,2,……,n$. Also suppose that component *i* has weight w_i , $i = 1,2,……,n$ and each w_i is a positive integer and $p_i = \Pr(g_i = 1)$ is the reliability of the component *i* and $q_i = 1 - p_i$, $i = 1,2,…..,n$. Define the vectors

$$Y = (Y_1, Y_2, ..., Y_n),$$
$$K = (K_1, K_2, ..., K_n),$$
$$\theta^{(1)} = (\theta_1^{(1)}, \theta_2^{(1)}, ..., \theta_n^{(1)}, ...),$$
$$\theta^{(0)} = (\theta_1^{(0)}, \theta_2^{(0)}, ..., \theta_n^{(0)}, ...),$$

where

$$Y_i = w_i(1 - g_i),\ i = 1,2,......,n$$

$$K_i = w_i g_i,\ i = 1,2,........,n$$

and

$$\theta_{i+1}^{(0)} = \begin{cases} \theta_i^{(0)} + Y_{i+1} & \text{if} \quad Y_{i+1} \neq 0 \\ 0 & \text{if} \quad Y_{i+1} = 0 \end{cases}$$, $i = 0, 1, 2,....., n, n+1,....$

$$\theta_{i+1}^{(1)} = \begin{cases} \theta_i^{(1)} + K_{i+1} & \text{if} \quad K_{i+1} \neq 0 \\ 0 & \text{if} \quad K_{i+1} = 0 \end{cases}$$, $i = 0, 1, 2....., n, n+1,.......$

here $Y_{n+r} = Y_r$ and $K_{n+r} = K_r$

$$\theta_0^{(0)} = \theta_0^{(1)} = 0.$$

From here we have

$\sum_{i=1}^{n} Y_i$ = Total weight of the failed components.

$\sum_{i=1}^{n} K_i$ = Total weight of the working components.

From the definition of Y_i and K_i it is clear that these are the elements of the set $\{0, w_i\}$ and also $\theta_i^{(1)}, \theta_i^{(0)} \in \left\{0, w_i, w_{i-1} + w_i, \ldots\ldots, \sum_{j=1}^{i} w_j\right\}$, i = 1,2,3,....,n, n+1,....., $w_0 = 0$.

For the existence of the circular weighted-(n, f, k, $\boldsymbol{w}$): F system all the elements of the sequence $\left\{\theta_i^{(0)}, i \geq 1\right\}$ must be less than k and $\sum_{i=1}^{n} Y_i$ must be less than f. Thus reliability of circular weighted-(n, f, k, $\boldsymbol{w}$): F system is given as:

$$CR_w(n, f, k : F) = P\left\{\theta_i^{(0)} < k, \ldots\ldots, \theta_n^{(0)} < k, \theta_{n+1}^{(0)} < k \ldots\ldots, \sum_{i=1}^{n} Y_i < f\right\},$$

The unreliability of the circular weighted-(n, f, k, $\boldsymbol{w}$): G system can be obtained as follows:

$$CF_w(n, f, k : G) = P\left\{\theta_i^{(1)} < k, \ldots\ldots, \theta_i^{(1)} < k, \theta_{n+1}^{(1)} < k \ldots\ldots, \sum_{i=1}^{n} K_i < f\right\},$$

Now we present some formulae to calculate the reliability (unreliability) of the above discussed systems.

The u- function for component 1 is given as:

$$U_1(z) = \sum_{g_1=0}^{1} p_1^{g_1} (1 - p_1)^{1-g_1} z^{g_1, Y_1, K_1}$$

The u- function for component 1 and 2 is given as:

$$U_{1,2}(z) = \sum_{g_2=0}^{1} \sum_{g_1=0}^{1} \left[\prod_{i=1}^{2} p_i^{g_i} (1 - p_i)^{1-g_i}\right] z^{(g_1, g_2),\ \sum_{i=1}^{2} Y_i,\ \sum_{i=1}^{2} K_i}$$

The u- function for components 1, 2, 3.......and n is given as:

$$U_{1,2,\ldots\ldots,n}(z) = \sum_{g_n=0}^{1} \sum_{g_{n-1}=0}^{1} \ldots\ldots\ldots\ldots \sum_{g_1=0}^{1} \left[\prod_{i=1}^{n} p_i^{g_i} (1-p_i)^{1-g_i}\right] z^{(g_1, g_2, \ldots\ldots, g_n),\ \sum_{i=1}^{n} Y_i,\ \sum_{i=1}^{n} K_i}$$

4. EXAMPLE

Suppose a system consists of 4 elements arranged circularly and weight of the elements are given by vector $\boldsymbol{w}$ = (3, 3, 2, 1). Let P_i is the probability of the success of the element i where i =1, 2, 3, 4. Suppose probabilities of success of elements 1, 2, 3 and 4 are 0.6, 0.5, 0.7, and 0.8 respectively. Diagrammatically it can be shown as depicted in Fig. 1.

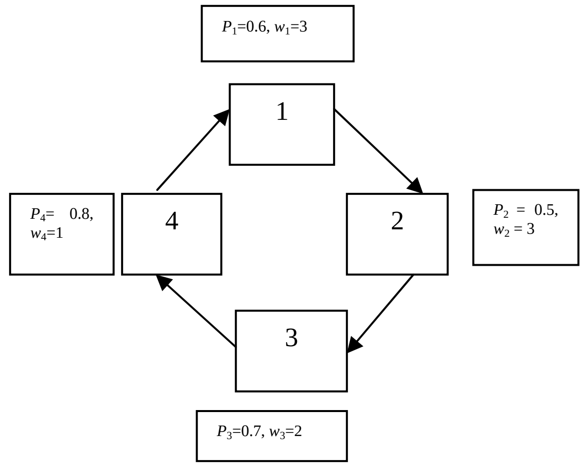

Fig. 1: Circular (4, *f, k, w*): *F* (*G*) system

The UGF of the circular (4, f, k, w): F (G) is obtained as:

$$\begin{aligned} U_{1,2,3,4}(z) = {} & p_1p_2p_3p_4\, z^{(1,1,1,1),0,w_1+w_2+w_3+w_4} + p_1p_2p_3q_4\, z^{(1,1,1,0),w_4,w_1+w_2+w_3} + p_1p_2q_3p_4\, z^{(1,1,0,1),w_3,w_1+w_2+w_4} \\ & + p_1p_2q_3q_4\, z^{(1,1,0,0),w_3+w_4,w_1+w_2} + p_1q_2p_3p_4\, z^{(1,0,1,1),w_2,w_1+w_3+w_4} + p_1q_2p_3q_4\, z^{(1,0,1,0),w_2+w_4,w_1+w_3} \\ & + p_1q_2q_3p_4\, z^{(1,0,0,1),w_2+w_3,w_1+w_4} + p_1q_2q_3q_4\, z^{(1,0,0,0),w_2+w_3+w_4,w_1} + q_1p_2p_3p_4\, z^{(0,1,1,1),w_1,w_2+w_3+w_4} \\ & + q_1p_2p_3q_4\, z^{(0,1,1,0),w_1+w_4,w_2+w_3} + q_1p_2q_3p_4\, z^{(0,1,0,1),w_1+w_3,w_2+w_4} + q_1p_2q_3q_4\, z^{(0,1,0,0),w_1+w_3+w_4,w_2} \\ & + q_1q_2p_3p_4\, z^{(0,0,1,1),w_1+w_2,w_3+w_4} + q_1q_2p_3q_4\, z^{(0,0,1,0),w_1+w_2+w_4,w_3} + q_1q_2q_3p_4\, z^{(0,0,0,1),w_1+w_2+w_3,w_4} \\ & + q_1q_2q_3q_4\, z^{(0,0,0,0),w_1+w_2+w_3+w_4,0} \end{aligned}$$

Let us discuss following special cases pertaining to the considered system:

***Case* 1:** when f = 4 and k = 3, i.e. the total weight of the failed components is 4 and the total weight of the consecutive failed components is 3, then

$$CR_w(4,4,3:F) = p_1p_2p_3p_4 + p_1p_2p_3q_4 + p_1p_2q_3p_4 + p_1p_2q_3q_4 + p_1q_2p_3p_4 + q_1p_2p_3p_4$$

$$CF_w(4,4,3:G) = p_1q_2q_3q_4 + q_1p_2q_3q_4 + q_1q_2p_3p_4 + q_1q_2p_3q_4 + q_1q_2q_3p_4 + q_1q_2q_3q_4$$

***Case* 2:** when f = 5 and k = 3, then

$$\begin{aligned} CR_w(4,5,3:F) = {} & p_1p_2p_3p_4 + p_1p_2p_3q_4 + p_1p_2q_3p_4 + p_1p_2q_3q_4 + p_1q_2p_3p_4 + p_1q_2p_3q_4 \\ & + q_1p_2p_3p_4 + q_1p_2p_3q_4 \end{aligned}$$

$$\begin{aligned} CF_w(4,5,3:G) = {} & p_1q_2q_3p_4 + p_1q_2q_3q_4 + q_1p_2q_3p_4 + q_1p_2q_3q_4 + q_1q_2p_3p_4 + q_1q_2p_3q_4 \\ & + q_1q_2q_3p_4 + q_1q_2q_3q_4 \end{aligned}$$

***Case* 3:** when f = 6 and k = 3, then

$$\begin{aligned} CR_w(4,6,3:F) = {} & p_1p_2\ p_3p_4 + p_1p_2p_3q_4 + p_1p_2q_3p_4 + p_1p_2q_3q_4 + p_1q_2p_3p_4 + p_1q_2p_3q_4 + p_1q_2q_3p_4 \\ & + q_1p_2p_3p_4 + q_1p_2p_3q_4 + q_1p_2q_3p_4 \end{aligned}$$

$$\begin{aligned} CF_w(4,6,3:G) = {} & p_1q_2q_3p_4 + p_1q_2q_3q_4 + q_1p_2q_3p_4 + q_1p_2q_3q_4 + q_1q_2p_3p_4 + q_1q_2p_3q_4 \\ & + q_1q_2q_3p_4 + q_1q_2q_3q_4 + p_1q_2p_3q_4 + q_1p_2p_3q_4 \end{aligned}$$

***Case* 4:** when f = 4 and k = 4, then

$$CR_w(4,4,4:F)=p_1p_2p_3p_4+p_1p_2p_3q_4+p_1p_2q_3p_4+p_1p_2q_3q_4+p_1q_2p_3p_4+q_1p_2p_3p_4$$

$$CF_w(4,4,4:G)=p_1q_2q_3q_4+q_1p_2q_3q_4+q_1q_2p_3p_4+q_1q_2p_3q_4+q_1q_2q_3p_4+q_1q_2q_3q_4$$

***Case* 5:** when $f = 4$ and $k = 5$, then

$$CR_w(4,4,5:F)=p_1p_2p_3p_4+p_1p_2p_3q_4+p_1p_2q_3p_4+p_1p_2q_3q_4+p_1q_2p_3p_4+q_1p_2p_3p_4 +q_1p_2p_3q_4$$

$$CF_w(4,4,5:G)=p_1q_2q_3p_4+p_1q_2q_3q_4+q_1p_2q_3q_4+q_1q_2p_3p_4+q_1q_2p_3q_4 +q_1q_2q_3p_4+q_1q_2q_3q_4$$

***Case* 6:** when $f = 4$ and $k = 6$, then

$$CR_w(4,4,6:F)=p_1p_2p_3p_4+p_1p_2p_3q_4+p_1p_2q_3p_4+p_1p_2q_3q_4+p_1q_2p_3p_4 +q_1p_2p_3p_4+q_1p_2p_3q_4+p_1q_2q_3p_4$$

$$CF_w(4,4,6:G)=p_1q_2q_3p_4+p_1q_2q_3q_4+q_1p_2q_3q_4+q_1q_2p_3p_4+q_1q_2p_3q_4 +q_1q_2q_3p_4+q_1q_2q_3q_4+q_1p_2p_3q_4$$

Now substituting the assumed reliabilities p_1= 0.6, p_2 = 0.5, p_3 = 0.7, p_4 = 0.8 of the components 1, 2, 3 and 4 respectively in above six cases, we get values of the reliability (unreliability) of the systems as:

Table 2: Reliability (unreliability) of circular (*n, f, k, w*): *F* (*G*) system in different cases

Cases	(*n, f, k*)	CR*w* (*n, f, k: F*)	CF*w* (*n, f, k: G*)
1	(4, 4, 3)	0.580	0.230
2	(4, 5, 3)	0.650	0.350
3	(4, 6, 3)	0.794	0.434
4	(4, 4, 4)	0.580	0.230
5	(4, 4, 5)	0.608	0.302
6	(4, 4, 6)	0.680	0.330

The effect of *f* and *k* on circular weighted (4, *f, k, w*): *F* systems are shown in Fig. 2 and Fig. 3 respectively.

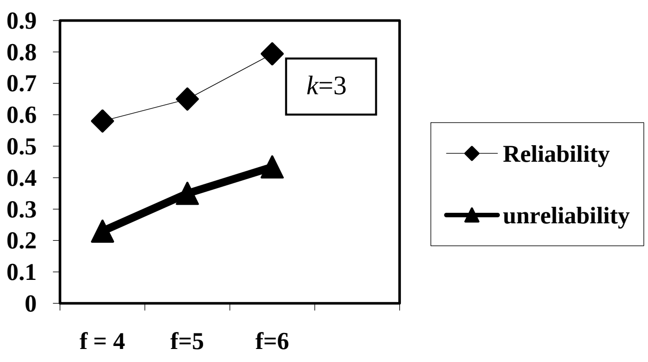

Fig. 2: Effect of *f* on reliability (unreliability) of circular weighted-(4*, f, k, w*): *F*(*G*) system

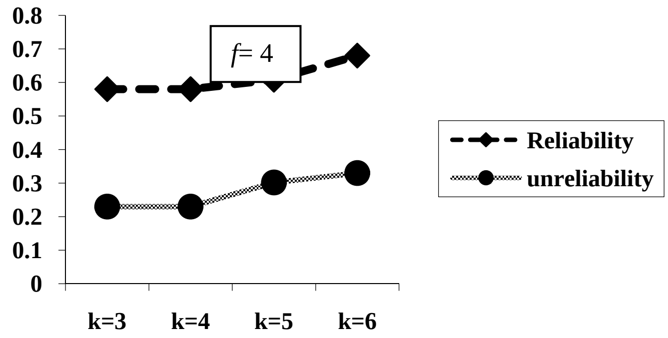

Fig. 3: Effect of *k* on reliability (unreliability) of circular weighted-(4*, f, k, w*): *F*(*G*) system

5. SENSITIVITY

Sensitivity of any element i with reliability p_i of the system is determined as follows:

$$S_i = \frac{\partial R}{\partial p_i}$$, where R is the reliability of system.

Sensitivities of the components in above discussed cases are given in Table 3 and effects of f and k on them are shown in Fig. 4 and Fig. 5 respectively.

Table 3: Sensitivities of elements in different cases

Cases	*S*1	*S*2	*S*3	*S*4
1	0.500	0.488	0.400	0.350
2	0.500	0.460	0.500	0.000
3	0.550	0.412	0.100	0.150
4	0.500	0.488	0.400	0.350
5	0.430	0.544	0.440	0.210
6	0.550	0.400	0.200	0.300

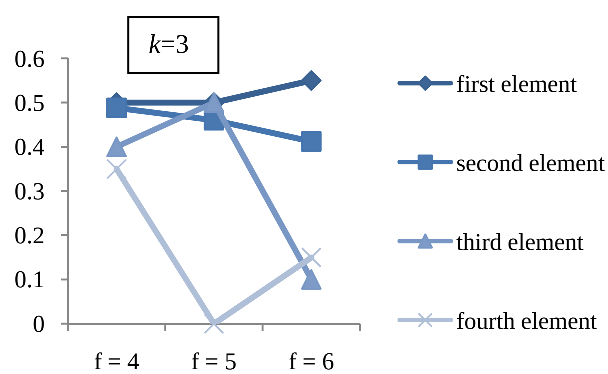

Fig. 4. Effect of *f* on sensitivities of different elements

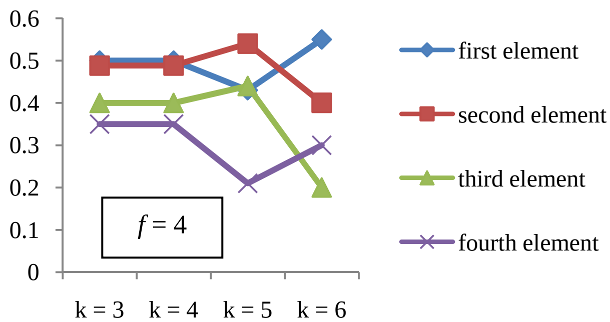

Fig. 5. Effect of *k* on sensitivities of different elements

6. CONCLUSIONS

Reliability and sensitivity evaluation of circular weighted-(n, f, k, $\boldsymbol{w}$): F (G) systems with independent and nonidentical components are proposed in this paper. We have obtained the reliability (unreliability) of these systems with the application of universal generating function approach. We assigned different conditions on the weight of the components and obtained reliabilities of the systems. Observation of study reveals that reliability (unreliability) increases with the increase in f and k when other values are kept constant. It is interesting to note that reliability (unreliability) of the system in cases when $k = 3$, $f = 4$ and $k = 4$, $f = 4$ are obtained to be exactly same.

We have also obtained the sensitivities of the components in various cases. Critical examination of the result shows:

Case 1: when $f = 4$ and $k = 3$, element 1 is most sensitive and element 4 is least.

Case 2: when $f = 5$ and $k = 3$, element 1 and 3 are most sensitive and element 4 is least.

Case 3: when $f = 6$ and $k = 3$, element 1 is most sensitive and element 3 is least.

Case 4: when $f = 4$ and $k = 4$, elements 1 is most sensitive and element 4 is least.

Case 5: when $f = 4$ and $k = 5$, element 2 is most sensitive and element 4 is least.

Case 6: when $f = 4$ and $k = 6$, element 1 is most sensitive and element 3 is least.

From the above discussion one can easily conclude that when f increases and k remains constant element 1 is the most sensitive and element 3 and 4 are least and when k increases and f remains constant element 1 and 2 are most sensitive and element 3 and 4 are least. The approach of reliability evaluation presented in this paper is very easy to handle.

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